A Dynamic Inconsistency Problem in PAYG: A Solution to the Turkish Puzzle
H. Yigit Aydede

Abstract: Because of the dynamic inconsistency problem in optimal policies of pay-as-you-go (PAYG) systems, parametric reforms tend to be unfair in terms of generational justice and could be inefficient in terms of optimal level of consumption. As long as there are adverse shocks, the planner has to decide on generational distribution of the financial burden in PAYG systems. In this paper we show that if intergenerational transfers are needed to keep the system in balance, any discretionary policy that allocates these transfers between the elderly and the young becomes dynamically inconsistent and the system moves toward being a Ponzi scheme. This may be part of the reason why the government has not been able to resist abusing the system by increasing its generosity for the elderly while expectations on social security wealth for new members have been declining for the last 40 years in Turkey.

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Keywords: Dynamic Inconsistency, Optimal Social Security, Social Security Wealth, Intergenerational Transfers, Life-Cycle Consumption.
JEL Codes: J1, H31, E21

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Turkey has one of the most generous social security systems in the Organization for Economic Co-operation and Development (OECD) region. According to the first comprehensive study conducted by the International Labour Organization (ILO) in 1996, the Turkish social security system will face an immense cumulative deficit by 2050, equaling an estimated to be 316 percent of gross domestic product (GDP). After two major economic crises in 1999 and 2001, Turkey has decided to reform the system with the help of the International Monetary Fund (IMF) and the World Bank. Since 1999, there have been four major reforms and countless amendments to social security. All reforms are parametric and phase in gradually so that the current generations will not be affected severely: despite the fact that the minimum age was increased to 65 in 2007, by 2020 many people will still be qualifying for retirement in their early 50s.

We are puzzled by the fact that while there has been a steady decline in expectations on social security wealth (SSW) for new entrants, the system has been growing to be more generous in the last 40 years. Between 1990 and 2003, the present value of the total resources used to finance the deficit of the social security system is almost equal to the total gross national product (GNP) created by Turkey in 2003. It is tempting to take this situation as a basic fact that the Turkish people foresee that the government may not be able to honor generous promises to current generations by increasing unpaid liabilities for coming generations and, therefore, reduce their expectations of the system. This explanation is not adequate unless the following question is answered: why is it the case that the government cannot resist exploiting the system so severely while the expectations have already been declining? It is plausible to think that, other than loose and opportunistic fiscal policies encouraged by developing country conditions, there may be reasons embedded in the very nature of pay-as-you-go (PAYG) systems that provide answers to this puzzle. We show that there is a dynamic inconsistency problem in optimal policies of PAYG systems so that parametric reforms tend to be unfair in terms of generational justice and could be inefficient in terms of the optimal level of consumption. As long as there are adverse shocks, the planner has to decide on generational distribution of the financial burden in PAYG systems. If intergenerational transfers are needed to keep the system in balance, any discretionary policy that allocates these transfers between the elderly and the young becomes dynamically inconsistent and the system moves toward being a Ponzi scheme. This can be part of the reason why parametric reforms gradually phase in and the generosity of the system continues to grow for the elderly in the last 40 years in Turkey.

The paper is organized as follows. Section 1 describes the generosity of the Turkish social security system and the declining trend in expected SSW. The dynamic inconsistency problem in PAYG policies is discussed in Section 2. Our brief interpretation of the results is presented in Section 3.

1. Turkish PAYG System
Old age security in Turkey, initiated after the second world war, consists of three state-
managed pension schemes\textsuperscript{2} that pay an earning-related defined benefit financed on a pay-as-you-go basis. In addition to its labor market distortions and redistribution to higher income groups, it is also financially insolvent. The lack of a minimum retirement age, which had been removed in 1992, has been the major factor for the financial imbalance—Turkey had retirement ages as young as 47 years of age in SSK (for wage earners) and 48 years of age in ES (for civil servants), the lowest in the world.\textsuperscript{3} Moreover, Turkey is the only country in the world that simultaneously had very low minimum contribution periods (in some cases as low as 10 years) and high replacement rates (90\% in SSK, 127\% in Bag-Kur, and 106\% in ES)\textsuperscript{4} with a lack of minimum retirement age before the 1999 reform (The World Bank Country Economic Memorandum, 2000).

The weak link\textsuperscript{5} between contributions and benefits before 1999 created an incentive for workers to declare the earnings base for paying a premium at a lower value. High informal employment rates due to relatively high statutory contribution rates, the lack of automatic indexation of the contribution ceiling\textsuperscript{6} under high inflation conditions, declining labor force participation rates, and the low premium collection rates because of administrative inefficiencies worsened the already financially imbalanced system. As a result, the system became a major fiscal burden, damaging Turkey’s macroeconomic stability. Coupled with other structural problems in the economy, the severe financial crisis early in 1999 forced the Turkish government, led by the World Bank, to reform the impaired social security system in August 1999. This parametric reform was intended to achieve actuarial balance of the PAYG system in the medium term and to reduce pressure on the borrowing needs of the government.

\begin{table}[h]
\centering
\caption{Pension and Income for Selected Countries in OECD (2002, in US dollars)}
\begin{tabular}{lcccccccc}
\hline
 & Per Capita & Average & Replacement & Gross & Relative & Relative \\
 & GNI & Wage & Rate & Pension & Pension & Pension \\
 & & & & Wealth & Wealth & Level \\
\hline
Hungary & 5,100 & 4,187 & 75.4 & 55,000 & 11.7 & 72.2 \\
Slovak Republic & 4,080 & 3,031 & 48.6 & 27,000 & 7.9 & 47.9 \\
Czech Republic & 5,880 & 6,306 & 44.4 & 47,000 & 6.9 & 41.7 \\
Mexico & 5,950 & 6,180 & 36.0 & 28,000 & 4.5 & 35.7 \\
Poland & 4,680 & 6,456 & 56.9 & 51,000 & 7.7 & 55.5 \\
Turkey & \textbf{2,510} & \textbf{6,571} & \textbf{87.2} & \textbf{74,000} & \textbf{10.3} & \textbf{81.3} \\
United Kingdom & 25,560 & 29,133 & 37.1 & 172,000 & 5.5 & 37.1 \\
United States & 35,430 & 32,360 & 38.6 & 183,000 & 5.2 & 36.5 \\
Canada & 22,610 & 24,756 & 42.5 & 163,000 & 6.1 & 39.9 \\
OECD Average & & & 56.9 & \textbf{202,367} & \textbf{8.7} & \textbf{55.4} \\
\hline
\end{tabular}
\end{table}

\textsuperscript{2} Social Insurance Institution (Sosyal Sigortalar Kurumu — SSK) for wage earners in private and public sectors, Bag-Kur (BK) for self-employed individuals and farmers, and Retirement Fund (Emekli Sandigi — ES) for civil servants.

\textsuperscript{3} The average minimum retirement ages for OECD countries are 64.4 for men and 63.9 for women in 2002 (OECD 2005).

\textsuperscript{4} These rates reflect 2004 values and are taken from Proposal for Reform in the Social Security System (2004).

\textsuperscript{5} In SSK, pensions were linked to wages paid in the last five years and the same link was even worse in ES and Bag-Kur: only the last year’s wages were used to calculate pensions.

\textsuperscript{6} The ceiling on wages subject to social security contribution in the fall of 1995 actually fell below the minimum wage. The World Bank Report (2000, 2003).

The most fundamental characteristics of the Turkish social security system have been its generosity (relative to income) due to very young average retirement ages (in some cases 38 for women and 47 for men⁷), low minimum contribution periods, and high replacement rates. As summarized in the table above, according to new research by the OECD (2005), a new entrant in Turkey has $74,000 average gross pension wealth with $2,510 per capita gross national income, compared with $183,000 and $35,430, respectively, for the US in 2002. In addition, Turkey’s gross replacement rate (87.2%) is the highest in the OECD region, far above the 56.7% average.⁸

However, this generosity is not sustainable. The total deficit financed by public borrowing (and seigniorage) is almost 4.5% of GDP in 2004. Between 1990 and 2003, the present value of the total resources used to finance the deficit of the social security system is almost equal to the total GNP created by Turkey in 2003.⁹ In other words, the generosity of the system has been financed by unpaid future liabilities, which does not even include the promises made by the government to unborn generations.¹⁰ The system’s generosity and unsustainable financial structure can be seen in Table 2 below, which uses the parameters of the system in 1996. The projected deficit for the system in 2005 underestimated the actual by almost 1% of GDP and that deficit projected for 2010 was realized by 2004.

### Table 2: Main Results of the Actuarial Projections

<table>
<thead>
<tr>
<th>Year</th>
<th>Reverse Dependency Ratio</th>
<th>Replacement Rate</th>
<th>Deficit % of GDP</th>
<th>Cumulative Deficit % of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>55%</td>
<td>95%</td>
<td>1.80%</td>
<td>1.80%</td>
</tr>
<tr>
<td>2000</td>
<td>65%</td>
<td>96%</td>
<td>2.70%</td>
<td>19.70%</td>
</tr>
<tr>
<td>2005</td>
<td>71%</td>
<td>97%</td>
<td>3.40%</td>
<td>32.10%</td>
</tr>
<tr>
<td>2010</td>
<td>80%</td>
<td>99%</td>
<td>4.30%</td>
<td>50.20%</td>
</tr>
<tr>
<td>2020</td>
<td>90%</td>
<td>100%</td>
<td>5.60%</td>
<td>97.30%</td>
</tr>
<tr>
<td>2030</td>
<td>96%</td>
<td>100%</td>
<td>7.00%</td>
<td>153.90%</td>
</tr>
<tr>
<td>2050</td>
<td>115%</td>
<td>101%</td>
<td>10.10%</td>
<td>316.00%</td>
</tr>
</tbody>
</table>

**Sources:** ILO, IMF, OECD, the World Bank, and the Ministry of Labor Affairs.

In order to calculate the expected SSW for a typical new member of the system in each age cohort, we use the SSW simulations done by Aydede (2007), which follow two major steps.

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⁷ *Türk Emeklilik Sisteminde Reform* (The Reform in Turkish Pension System), TUSIAD, November 2004, P.108
⁸ Excluding Luxembourg which is an outlier with 101.9% gross replacement rate.
¹⁰ The longevity, 69 years, in 2005 is expected to be 79 years in 2045 in Turkey.
as explained below\textsuperscript{11}:

**Present Value of Future Benefits (SSWG):**

- If an individual at the age of \((a)\) in year \((t)\) survives to the retirement age \((ra)\), and if his current wage (or income that the old-age security is based on), \(W_{(a,t)}\), grows at a constant rate of growth, \(g\), then he will have an income at the retirement age of \(ra\):

\[
W_{t+ra-a} = W_t(1+g)^{(ra-a)}
\]  

(1)

- In order to calculate the first annuity for this individual, we use a benefit factor, \((bf)\), which is basically the ratio of his first annuity to his last wage (or to his insurable income if he is among self-employed individuals).

- Given the benefit factor, the individual will be entitled to his first annuity at \(ra\), which is

\[
B_{(a,t)} = bf_t W_t (1+g)^{(ra-a)}
\]  

(2)

- If we further assume that real annuities will grow after \(ra\) by \(ga\) until the age of death \((da)\), given the survival probabilities \((S_{(i,j)})\) for that particular individual, the actuarial present value of future annuities \((PVA)\) can be calculated at \(ra\), where \(S_{(i,j)}\) presents the probability of living at least to the age of \(j\), given that the person lived to the age \(i\).

With the personal discount rate \((d)\) for future real incomes, Equation (2) becomes at the age of \(ra\):

\[
PVA_{ra} = \sum_{da \geq n \geq ra} S_{(ra,n)} B_{(a,t)} (1+g)^{(n-ra)} (1+d)^{-(n-ra)}
\]  

(3)

- At time \((t)\), after substituting \(B_{(a,t)}\) (Equation 2) into Equation 3, the person has the following expected present value of benefits at the age of \(a\):

\[
SSWG_{(a,t)} = bft W_t S_{(a,ra)} \left[ (1 + g) / (1 + d) \right]^{(ra-a)} \sum_{da \geq n \geq 65} S_{(ra,n)} \left[ (1 + g) / (1 + d) \right]^{(n-ra)}
\]  

(4)

This also includes survival probabilities between the age of retirement \((ra)\) and the current age \((a)\) at time \((t)\).

**Present Value of Future Contributions (SSTX):**

- If the same individual at the age of \((a)\) in year \((t)\) survives to age \(ra\), and if his current wage (or income that the old-age security is based on), \(W_{(a,t)}\), grows at a constant rate of growth, \(g\), then the expected present value of all his future contributions until age \(ra\) can be calculated as follows:

\[
SSTX_{a,t} = \sum_{m=a}^{ra} S_{a,m} \theta_{t+m-a} W_t \left[ (1 + g) / (1 + d) \right]^{m-a}
\]  

(5)

\textsuperscript{11} SSW series were first calculated by Feldstein (1974) and criticized by Leimer and Lesnoy (1980, 1982).
where $\theta$ is the ratio of social security taxes to his wages through his working years, and the person expects that at the age $m$ he will pay a tax of $T_{t+m-a} = \theta_{t+m-a} W_t (1+g)^{m-a}$. As noticed in Equation (5), SSTX treats contributions paid in the past as “sunk.” This could be easily adjusted by adding the present value of all paid contributions to the equation.

Equations (4) and (5) above are the fundamental equations in calculating the expected SSW, which is the difference between $SSWG$ and $SSTX$. We take the retirement part of the programs and exclude other attached benefits such as health care. Other than ours, there is only one study done by Brook and Whitehouse (2006) showing the distributional characteristics of the Turkish PAYG system calculated by actuarial realizations. Their simulation is based on the eligibility of a generic worker to specific annuitization rules and minimum age requirements of the system for different age cohorts. Our cohort-specific SSW and implicit rate of return (IRR) values represent perceptions that each new entrant in each age cohort may have from social security and not the actuarial realization that a new entrant may end up with at the end of his/her life. Therefore, the SSW that a representative individual in each age cohort might expect can be different than what actually he/she will be entitled at the end. In finding expected SSW and the implicit rate of return for a representative new member of the system for different years and genders, we look at a typical worker entering the labor force at the age of 17 and expecting to retire at the age of 55. Unlike the Brook and Whitehouse’s study (2006), we are rather interested in finding the variation in the expected SSW among new entrants in different years. Values based on actuarial realization, such as pension wealth (PW) and benefit-cost ratio (BCR), vary only by the eligibility of each individual to the specifics of the program. However, since there were no effective minimum age requirements before 2000 in Turkey, each hypothetical individual in PW and BCR calculations has the same retirement age assumption, which had remained the same until 2000. Therefore, actuarial realizations of PW and BCR before 2000 cannot reveal any information about the difference in perceptions of the SSW ‘deal’ for different age cohorts. In this sense, our simulations are not directly sensitive to parametric changes in the system and represent aggregate expectations of new members about social security wealth and not actuarial values to which different age cohorts are entitled. In our simulations, we also assume that people with the available information take the current factors (benefit and tax) as their expectations. The values of SSW and IRR are directly linked to these factors. However, when we use moving averages of tax and benefit factors, the trend in IRR becomes very similar to one in Figures 1 below.

**Figure 1: Trend in Expected IRR for New Members** (Males)

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12 SSW series are used to identify generational fairness of social security systems. Therefore, they are mostly calculated based on actuarial realizations. See Boskin et al. (1985), Steuerle and Bajika (1974), and Meyer and Wolff (1987).

13 See Aydede (2007) for the details of assumptions and the simulation methods.
The trend in IRR for both genders\textsuperscript{14} shows that new members of the system in every succeeding age cohort anticipate a lower return for their social security contributions. Even though our simulation indicates a short period of rise in expectations between 1990 and 1996, this escalation is mainly caused by populist policies during a politically unstable period. There are four parametric reforms and several amendments after 1999 that increase the required minimum retirement age for new participants and adjusts the annuitization. In addition to the fact that all these changes have occurred in as short a period as four years, many initial amendments have been canceled by the Supreme Court and replaced by new ones. Consequently, it is hard to speculate about how people formed their expectations in this “confusing” period about social security. Although we incorporate the changes in age requirements into our simulations by increasing the retirement age from 55 to 58 and later to 60 for new members, we do not adjust the way that benefits are calculated in our simulations.\textsuperscript{15}

To understand the trend in the generosity for a typical retiree in each age cohort for the last 40 years in Turkey, we calculated the growth of per-retiree annuities in real terms, as Figure 2 below shows. If the entire period is taken, the average retirement benefit payment increases 0.5\% each year in real terms. However, the same average for the last 24 years (1980-2003) is 2.4\%, which shows that retirees have not been falling behind in receiving their share of the generosity from the system\textsuperscript{16}.

\textbf{Figure 2: Growth in Annuities} (with 1987 Prices)

\textsuperscript{14} The IRR values are significantly better for females. The fundamental reason for this difference is that females have significantly higher survival rates.

\textsuperscript{15} The new calculation method for annuities is rather complex. Since the change has been in effect after 2001, we only changed the minimum retirement age for new members and kept our fundamental functions the same to calculate first annuities based on benefit factor expectations. This negligence has very little effect on the aggregate SSW series since the change covers the subsequent three years after 2001 and the share of new members in the total covered individuals is less than 5\%.

\textsuperscript{16} However, if we remove the outlier growth (60\%) in 1996, the average approaches zero.
Our results show that while the generosity of the system has been growing, there was also a significant decreasing trend in SSW expectations for new members of the Turkish PAYG system in the last 40 years. In the following section, we show that this decreasing trend in expectations creates an irresistible incentive to government to exploit the system due to the dynamic inconsistency problem in optimal PAYG policies.

2. Dynamic Inconsistency of Optimal Policies in PAYG

The concept of optimal social security is not new and has been discussed extensively in the literature since Samuelson’s prominent paper appeared in 1958. When a PAYG system is introduced to the economy due to private myopia, the question becomes whether there is an optimal policy for the social planner, which leads to “golden-rule equilibrium that maximizes lifetime wellbeing of every subsequent generation” (Samuelson 1975, p.539). This question is put differently today: given demographic challenges and negative trends in labor force participation, what are the optimal policies of PAYG in terms of generational fairness and economic growth? In the absence of these adverse shocks, optimal benefits and taxes guide generational transfers from workers to retirees that are neutral in terms of present value of anticipated lifetime wealth for individuals, except for the startup generation. When the economy grows more than what the productive capital earns, optimal PAYG policies can be welfare improving as well. Therefore, the usual optimality condition, i.e. the equality of the interest rate to the growth rate of population plus productivity, can be achieved by policies, that are sustainable, time-consistent, and predictable. When there are adverse shocks in the system, however, they are not predictable and transfers needed to balance the system may not be neutral anymore. Even if they are optimal, the government’s policies in PAYG might have effects on consumption and saving depending on the predictability of these policies by individuals. Our concern is not to find the new parameters, which lead to the golden-rule after recurrent shocks, but to question which structure of PAYG policies can be appropriate in these transition paths.

To analyze outcomes of optimal policies in PAYG systems, we use a simple two-period overlapping generation model where identical individuals are endowed by $w$ for their inelastic labor supply and save $(s)$ in the first period, and retire and spend $(c)$ their saving in the second period. To simplify further, we assume that there is no uncertainty (in lifetime), liquidity constraint, bequest, growth in endowment, and that the real interest rate $(r)$ is equal to the
personal discount rate ($\rho$). The only function of the government is to organize a balanced PAYG system where the young, $y$, pay taxes, $tx$, and the old, $o$, receive benefits, $b$. We bring intergenerational transfers into this model by introducing an unexpected adverse permanent demographic shock ($\theta$) to the system, at time $t-1$ as follows.

$$R_{t-1}b_{t-1} = L_{t-1}tx_{t-1} \Rightarrow b_{t-1} = \lambda_{t-1}tx_{t-1} \Rightarrow b_{t-1}^* = (\lambda_{t-1} - \theta_{t-1})tx_{t-1} \Rightarrow b_{t-1} > b_{t-1}^*$$  \hspace{1cm} (1)

where $R$ and $L$ stand for numbers of retirees and workers respectively. The (reverse) dependency ratio, $\lambda$, is $L/t$. Since taxes are fixed, transfers are used to balance the system. We want $R_t$ to be equal to $L_{t-1}$ to avoid heterogeneity among retirees, thus we assume that adverse shocks are due to negative changes in fertility.\(^\text{17}\) Consequently, $\lambda_{t-1} = 1 + n_{t-1}$ and changes in population will be introduced to the system by adverse shocks such that $\lambda_t = 1 + n_t = \lambda_{t-1} - \theta_{t-1}$. As long as shocks are persistent, the system needs transfers ($tr_t$) for each succeeding cohort as follows.

$$b_{t-1} = (\lambda_{t-1} - \theta_{t-1})tx + tr_{t-1}$$ \hspace{1cm} (2a)

$$tr_{t-1} = \delta_{t-1} \lambda_{t-1}tx$$ \hspace{1cm} (2b)

$$tr_s = tx\delta \sum_{i=t-1}^s \theta_i$$ \hspace{1cm} (2c)

$$0 \leq \delta \leq 1$$ \hspace{1cm} (2d)

where $\delta$ is determined by the government such that as it approaches one, the full burden ($tx\theta$) will be born by the next generation of workers. Our setting is different from the conventional way of thinking about generational transfers caused by PAYG. First, as we explain below, when there is a shock to the system, zero distribution ($\delta_{t-1} = 0$) may not be optimal. Second, even if expectations on future shocks are zero, every succeeding generation may have different expectations on sharing a part of the initial burden as stated by (2c) above.

From the individual’s perspective, generational transfers are recognized when they are required to pay $tr$ and form their expectations in the first period on whether or not the system will be fair in the second period. However, which generation transfers, and how much, will be determined by the government’s generational policy in the second period, which can be either discretionary or governed by a rule. Hence, the total consumption at time $t$ will be affected by three factors: (1) the expectation of the old formed at time $t-1$, (2) the government’s generational policy at time $t$, and (3) the expectation of the young at time $t$.

Formally, every young person who is required to pay taxes, $tx$, and an additional transfer, $tr_{t-1}$ faces the following problem:

$$Max_c u(c_{t-1}) + (1 + \rho)^{-1} E_{t-1} u(c_t)$$

subject to

\(^{17}\) If we include shocks to labor force participation and longevity, the aggregation becomes more complex.
\[ c_{st-1} = w - tx - tr_{t-1} - s_{t-1} \]

and

\[ E_{t-1}c_{st} = (1 + r)s_{t-1} + E_{t-1}(b_t) . \]

With usual assumptions in the utility function, the young at time \( t-1 \) solve this problem with the following values:

\[ c_{st-1} = \frac{1+r}{2+r} \left[ w - tx - tr_{t-1} + E_{t-1}b_t (1+r)^{-1} \right] \]  \hspace{1cm} (3a)

\[ s_{t-1} = \frac{1}{2+r} \left[ w - tx - tr_{t-1} - E_{t-1}b_t \right] . \]  \hspace{1cm} (3b)

Expectations in the first period will determine the saving. The young can expect a fair (\( E_{t-1}b_t/(1+r) - tx_{t-1} - tr_{t-1} \geq 0 \)) or unfair PAYG (\( E_{t-1}b_t/(1+r) - tx_{t-1} - tr_{t-1} < 0 \)). The government’s generational policy will determine \( b_t \) and the consumption in the second period:

\[ c_{st} = (1 + r)s_{t-1} + b_t = \frac{1+r}{2+r} \left[ w - tx - tr_{t-1} - E_{t-1}b_t \right] + b_t . \]  \hspace{1cm} (3c)

Moreover, the young’s consumption at time \( t \) is also determined by the expectation on social security as shown below.

\[ c_{st} = \frac{1+r}{2+r} \left[ w - tx - tr_t + E_{t-1}b_{t+1} (1 + r)^{-1} \right] . \]  \hspace{1cm} (3d)

Given that all individuals recognize that the planner commits to a balanced-budget (2a), the young’s expectation on benefits at \( t-1 \) becomes:

\[ E_{t-1}b_t = (1 + n_t - E_{t-1} \theta_t)tx + (1 + n_t - E_{t-1} \theta_t)tr_t . \]  \hspace{1cm} (4a)

Using the fact that \( (1 + n_t) = (1 + n_{t-1} - \theta_{t-1}) \) and (2c), we obtain

\[ (1 + n_t - E_{t-1} \theta_t)tr_t = E_{t-1} \delta_t (\theta_{t-1} + E_{t-1} \theta_t)tx . \]  \hspace{1cm} (4b)

By (4a) we can express expected benefits in terms of taxes as follows;

\[ E_{t-1}b_t = tx(1 + n_{t-1}) - tx(1 - E_{t-1} \delta_t)(\theta_{t-1} + E_{t-1} \theta_t) . \]  \hspace{1cm} (4c)

Likewise, the transfer that the young pay at time \( t-1 \) is
\[ tr_{t-1} = \frac{\delta_{t-1} \theta_{t-1} tx}{1 + n_{t-1} - \theta_{t-1}}. \] (4d)

Substituting (4c) and (4d) into (3a) and assuming \( E_{t-1} \theta_{t} = 0 \), we obtain;

\[ c_{y_{t-1}} = \frac{1}{2 + r} \left[ (1 + r)w + (n_{t-1} - \theta_{t-1} - r)tx - \frac{(1 + r) \theta_{t-1} tx \delta_{t-1} + \theta_{t-1} tx E_{t-1} \delta_{t}}{1 + n_{t-1} - \theta_{t-1}} \right] \] (4e)

and

\[ c_{o_{t}} = (1 + r) \left[ w - tx - \frac{\delta_{t-1} \theta_{t-1} tx}{1 + n_{t-1} - \theta_{t-1}} - c_{y_{t-1}} \right] + b_{t} \] (4f)

where \( b_{t} \) is determined by the planner as follows:

\[ b_{t} = tx(1 + n_{t-1}) - tx(1 - \delta_{t})(\theta_{t-1} + \theta_{t}). \]

The optimal level of \( \delta \) will be determined by the planner’s objective function. If the social discount rate \((\beta)\) for future generations is high enough in this function, even in the absence of dynamic inefficiency with \( r < n_{t} \), an unexpected introduction of PAYG increases the total social welfare. This is because the gain for the old, who receive benefits without paying for it, can be weighted high enough to surpass the total loss for future generations. Therefore, as shown by Marini and Scaramozzino (1999), since an optimal social security critically depends on the level of social discount rate, the existence of PAYG can be justified entirely on ethical grounds. Likewise, when there are adverse shocks and the government is committed to balancing the system by transferring from the young to the old, the share of each transfer between generations will be determined by the social discount rate: the higher the discount rate is, the higher the burden shifted to next generations will be. If the time-consistent\(^\text{18}\) social welfare function is given by

\[ SW_{t-1} = \frac{1 + \beta}{(1 + \rho)(1 + n_{t-1} - \theta_{t-1})} u_{o}(c_{o_{t-1}}) + \sum_{s=t}^{\infty} \left( \frac{1 + n_{s-1} - \theta_{s-1}}{1 + \beta} \right)^{s-t} \left[ u_{y}(c_{y_{s-1}}) + (1 + \rho)^{-1} u_{o}(c_{o_{s}}) \right], \]

the net welfare effect of any generational policy depends on balancing the net gain to some generations against the loss to all future generations as expressed below:

\[ \frac{1 + \beta}{(1 + \rho)(1 + n_{t-1} - \theta_{t-1})} u'_{o}(c_{o_{t-1}}) = \sum_{s=t}^{\infty} \left( \frac{1 + n_{s-1} - \theta_{s-1}}{1 + \beta} \right)^{s-t} \left[ u'_{y}(c_{y_{s-1}}) + (1 + \rho)^{-1} u'_{o}(c_{o_{s}}) \right] \] (5)

\(^{18}\) The reverse discounting for the old is a necessary condition for a symmetric treatment between generations as explained by Calvo and Obstfeld (1988).
where $0 \leq \beta < \infty$, $(n_{t-1} - \theta_{t-1}) < \beta$, and $du / d\delta \equiv u'(c)$. From this condition, it is obvious that any policy with $\delta_{t-1} > 0$ can be feasible and that distributions from the young and unborn to current generations can be increased as $\beta$ goes up.

By our assumptions, since there is no substitution between leisure and consumption, transfers are nondistortionary. In addition, we do not have productive capital so that a reduction in saving has no effects on factor prices; hence, the change in consumption for each generation is the only consequence. In order to see the effect of a generational policy on consumption, we take two boundary examples where $\delta$ is zero and one. At time $t-1$, after the shock, the consumption of the old can be expressed as follows;

$$c_{st-1} = \frac{1 + r}{2 + r} \left[ \frac{r - n_{t-1}}{2 + r} \right] \theta_{t-1} - \theta_{t-1} t x (1 - \delta_{t-1})$$

(6a)

When the planner sets $\delta$ as zero, the old lose $\theta_{t-1} t x$. If the young at time $t-1$ take it as a consistent policy and set their expectations as $E_{t-1} \delta_{t} = 0$ and $E_{t-1} \theta_{t} = 0$, (4e) becomes

$$c_{st-1} = \frac{1 + r}{2 + r} \left[ \frac{r - n_{t-1}}{2 + r} \right] \theta_{t-1} t x - \frac{\theta_{t-1} t x}{2 + r}$$

(6b)

If we assume, for the moment, $r = 0$, all generations share the same burden: $\theta_{t-1} t x$. This result changes when $\delta$ is set to one. With this policy (6b) will be;

$$c_{st-1} = \frac{1 + r}{2 + r} \left[ \frac{r - n_{t-1}}{2 + r} \right] \theta_{t-1} t x - \frac{\theta_{t-1} t x}{2 + r}$$

(6c)

When the planner sets $\delta$ to one, the old gain $\theta_{t-1} t x$ but the loss for the young will depend on the parameters as given by the last term in (6c) above. In order to compare two policies in terms of their effects on consumption, we take the difference between (6b) and (6c) as follows:

$$c^{\delta=1}_{st-1} - c^{\delta=0}_{st-1} = \frac{\theta_{t-1} t x}{2 + r} \left[ \frac{n_{t-1} - \theta_{t-1} - r}{1 - n_{t-1} - \theta_{t-1}} \right].$$

As expected, a policy with $\delta > 0$ will be welfare improving if either $(n_{t-1} - \theta_{t-1}) \geq r$ or $\beta$ is sufficiently large enough so that (5) holds.

When there is a shock in the system at time $t-1$, the planner decides on $\delta_{t-1}$ that satisfies (5). Having faced $r_{t-1}$, the young set their expectations on $\delta_{t}$ at time $t-1$. After expectations are formed, the government chooses $\delta_{t}$, which determines $r_{t}$. When the policy is not binding, it is said to be time-consistent only if the same policy set at time $t$ is still optimal for the government at time $t+s$ after the expectations are formed, as expressed below:

$$\delta_{t+s}(t+s) = \delta_{t+s}(t)$$

The time-inconsistency in optimal policies of PAYG arises from the fact that the consumption in the second period depends on the saving decision made in the first period. Therefore, the
consumption at time \( t \) is a negative function of benefits expected at time \( t-1 \), as seen by (3c) given again below:

\[
c_{ot} = s_{t-1} + b_t = \frac{1}{2 + r}[w - E_{t-1}b_t - tx - tr_{t-1}] + b_t.
\]

If the young expect an unfair PAYG, they save more. Once they save more, because it is unexpected, a fairer PAYG policy by the government can generate a higher consumption at time \( t \) than when it is expected. In order to see the relationship between expectations and the government’s generational policy, we assume, for a moment, that \( r = n_{t-1} = 0 \) and \( E_{t-1}\theta_t = \theta_t = 0 \). With these assumptions (4f) becomes:

\[
c_{ot} = \frac{1}{2}(w - \theta_{t-1}tx) - \theta_{t-1}tx\left[\frac{1}{2}\frac{\delta_{t-1}}{1 - \theta_{t-1}} + \frac{1}{2}E_{t-1}\delta_t - \delta_t\right]. \tag{7a}
\]

And the consumption for the young at time \( t \) will be

\[
c_{yt} = \frac{1}{2}(w - \theta_{t-1}tx) - \frac{1}{2}\theta_{t-1}tx\left[\frac{\delta_{t}}{1 - \theta_{t-1}} - E_t\delta_{t+1}\right]. \tag{7b}
\]

Equation (7a) shows the relationship between expectations on the government policy \((E_{t-1}\delta_t)\) and the government’s decisions on the same generational policy \((\delta_{t-1}, \delta_t)\). Because \((n_{t-1} - \theta_{t-1}) < r\) by our assumption, the optimal policy for the government is to set \(\delta_{t-1}\) to zero at time \(t-1\). If this policy is binding or believed to be so by agents, every generation shares the same burden \((\theta_{t-1}tx)\) and the consumption for the old and the young becomes the same as follows:

\[
c_{ot} = c_{yt} = \frac{1}{2}(w - \theta_{t-1}tx).
\]

However, when expectations are set as \(E_{t-1}\delta_t = \delta_{t+1} = 0\), the policy announced at time \(t-1\), \(\delta(t-1) = 0\), turns out not to be optimal anymore. This is because, any policy, such that \(\delta_t(t) > E_{t-1}\delta_t\) has a positive effect on the consumption at time \(t\) as shown below:

\[
c_{ot} = \frac{1}{2}(w - \theta_{t-1}tx) + \theta_{t-1}tx\delta_t \tag{7c}
\]

\[
c_{yt} = \frac{1}{2}(w - \theta_{t-1}tx) - \frac{1}{2}\theta_{t-1}tx\left[\frac{\delta_t}{1 - \theta_{t-1}} - E_t\delta_{t+1}\right]. \tag{7d}
\]

And, therefore, it may be said that

\[\delta_t(t) \neq \delta_t(t-1).\]
As shown by (7c) and (7d), the net effect of the policy on the total consumption at time \( t \) depends on three factors: (1) aggregation of individual consumption functions; (2) the difference between marginal propensity to consume (MPC) of the old and the young; (3) expectations of the young on the future policy \( (E_t \delta_{t+1}) \). The numbers of retirees and workers will determine the total consumption at time \( t \). In our model, the MPC for the old is one while it is one-half for the young. However, in the existence of altruistic transfers, imperfect capital markets, and accidental bequests, the old may have a lower MPC, which may reduce positive effects of reneging the previous policy on consumption for the government. On the other hand, any positive correlation between the expectation of the young and the current policy at time \( t \) magnifies the increase in total consumption.

The time-inconsistency problem can be stated formally as follows. After normalizing \( R_t \) to 1, the government wants to maximize the following social welfare function at time \( t \):

\[
SW_t = (1 + \beta)R_t u_o(c_{ot}) + L_t u_y(c_{yt}) + R_{t+1}u_o(c_{ot+1}) + A_t \tag{8a}
\]

subject to (2c), (2d), and

\[
c_{yt-1} = \frac{1}{2+r} \left[ (1+r)w + (n - \theta_{t-1} - r)tx - \frac{(1+r)\theta_{t-1}tx\delta_{t-1}}{1+n_{t-1}-\theta_{t-1}} + \theta_{t-1}txE_{t-1}\delta_s \right],
\]

\[
c_{ot} = (1+r) \left[ w - tx - \frac{\delta_{yt-1}\theta_{t-1}tx}{1+n_{t-1}-\theta_{t-1}} - c_{yt-1} \right] + b_s.
\]

The last part of (8a), the sum of utilities for future generations after \( t+1 \), \( A \), represents the present value of utility indexes for all future generations calculated by the social discount rate.\(^{19}\) If the reverse discounting for the old at time \( t \) is omitted, the argument becomes

\[
SW_t = \lambda_t \{ u_{yt}(E_{t-1}, \delta_t) + u_{ot+1}(E_{t+1} \delta_{t+1}, \delta_{t+1}) \} + u_{ot}(E_{t-1} \delta_{t-1}, \delta_{t-1}) + A_t
\]

The planner chooses its policy variable, \( \delta_t \), that satisfies the following first-order condition:

\[
\lambda_t (u'_{yt} + u'_{ot+1}) \left[ \frac{dE_t \delta_{t+1}}{d\delta_t} + 1 \right] + u'_{ot} \left[ \frac{dE_{t-1} \delta_{t}}{d\delta_t} + 1 \right] + \frac{dA_t}{d\delta_t} = 0
\]

Following the argument pioneered by Kydland and Prescott (1977), in order for an optimal policy to be consistent, the effect of \( \delta_t \) on \( E_{t-1} \delta_{t} \) should be zero,\(^{20}\) which is not possible if the policy is not committed to a rule. The reason for this failure is the existence of the incentive for the planner to renge its generational policy after the young form their expectations. As long as the young have the knowledge of the fact that the planner has an incentive to apply \( \delta(t) \) higher than \( \delta(t-1) \), they set their expectations as to receive a fairer PAYG. Once the expectations at

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\(^{19}\) This is because present policies cannot affect expectations of unborn generations at time \( t \).

\(^{20}\) Or the marginal utility of the old \( (u'_{ot}) \) should be zero.
time $t-1$ are set to receive a fairer PAYG, any discretionary policy which reduces consumption by setting an unfair (or less fair) PAYG may not be feasible. As a result, the entire system moves toward a full-scale Ponzi scheme.

We omitted the reverse discount rate for the old. The increase in social welfare depends on how the welfare of the elderly is valued by society in the face of adverse shocks. If the society puts a higher value on the marginal gain of the old from one dollar transfer than the marginal loss of the young from the same dollar, this incentive might be magnified.

3. Conclusion

As shown in the previous sections, every new member of the Turkish PAYG system has lower expectations in SSW while the system keeps its generosity for the elderly and cumulates unpaid liabilities for coming generations. The dynamic inconsistency problem could partly explain how the generosity of the system and the decreasing expectations in SSW exist together. It may also help us understand how, as people get older, they become proponents of the system, even if their life-time SSW is negative.

The time-inconsistency problem could partly explain how the generosity of the system and the decreasing expectations in social security wealth exist together. It may also help us understand how, as people get older, they become proponents of the system, even if their life-time social security wealth is negative. A decreasing trend in the expectations from PAYG creates an irresistible incentive for governments to abuse the system by reneging the policy set before. Therefore, as long as it is discretionary and not a rule, even in our partial equilibrium setting, a strictly neutral generational policy turns out to be unsustainable. In other words, the government employs populist policies to increase the consumption by giving ‘some’ to the old and promising ‘some’ to the young at time $t$. In this case, the young members may still expect unfair PAYG to the extent in which government tries its best to be ‘fair’. As the government approaches a hundred-percent fair PAYG and convinces people that it would honor its promises, the system grows to be a full-scale Ponzi scheme. As a result, an attempt to make PAYG fairer by implementing a new policy, which distributes the burden of adverse shocks among generations by setting $\delta_t$ lower than one, ends up with a higher level of consumption.

The latest reform passed in 2008 increases the minimum required age and makes annuitization rules more conservative than before. Since all rules phase in very gradually, only new members will be affected fully. Given that Turkey has had four major reforms and countless amendments in the last 6 years, the new reform can hardly be taken as a rule. If these new members set their expectations as previous generations, the same incentive to abuse the system will continue to exist for governments to come.
References


